Title: Understanding Data Shapes Through Skewness, Symmetry, and Kurtosis

Contents

Overview

This lesson aims to deepen students' understanding of statistical measures of data distribution, specifically skewness, symmetry, and kurtosis. Through a blend of interactive activities, discussions, and hands-on analysis, students will learn to interpret, create, summarize, and diagram the shape of data sets. This lesson is designed to engage students in applying these concepts to real-world data, enhancing their statistical literacy and analytical skills.

Learning Standards

CTE21.BMA.DA.1: Summarize and evaluate how statistics are used in business areas.

d. Demonstrate and explain the measure of shape of the data set using skewness, symmetry, and kurtosis

Objectives

Students will:

- Interpret the measure of shape of a data set using skewness, symmetry, and kurtosis.
- Create visual representations to demonstrate the measure of shape of data sets.
- Summarize their findings on the data set's shape using statistical terminology.
- Diagram the measure of shape using software tools or manual methods.

Duration

120 minutes (multiple class periods)

- Introduce (30 minutes)
- Explain (30 minutes)
- Inspire (60 minutes)

Materials

- QuantHaub Resources
  - Scavenger Hunt: What Does Variability, Skewness, and Kurtosis Describe About a Distribution?
  - Scavenger Hunt: Histograms and Density Plots

- Appendix Resources
  - Case Study: Explain Phase
  - Statistical Analysis and Careers Descriptions
  - Career Case Studies: Inspire Phase
  - How-To: Peer Review
  - Lesson Rubric

Background Knowledge

Students should have a foundational understanding of basic statistical measures such as:

Median: The median is the middle value in a list of numbers when they are arranged in order from smallest to largest. If there is an even number of values, the median is the average of the two middle numbers.

Mode: The mode is the number that appears most frequently in a list of numbers. It's the one that shows up the most times.

Mean: The mean is another word for the average. To find the mean of a set of numbers, you add up all the numbers and then divide by how many numbers there are.

Histogram: A histogram is a way to display data using bars on a graph. Each bar represents a range of values, and the height of the bar shows how many times values in that range appear in the data. It helps us see the distribution or pattern of data.

Density Plot: A density plot is a type of graph that shows the distribution of data points along a continuous scale, like a line or a curve. It gives you an idea of how closely packed or spread out the data is in different parts of the scale, helping to understand the shape of the data distribution.

Activities

Introduce (30 Minutes)

Present students with a real-world problem involving a data set (e.g., a company's sales data, population growth statistics, *or the provided data set*) that can be analyzed for skewness, symmetry, and kurtosis. Pose questions that pique their curiosity about how these measures can reveal insights into the data.

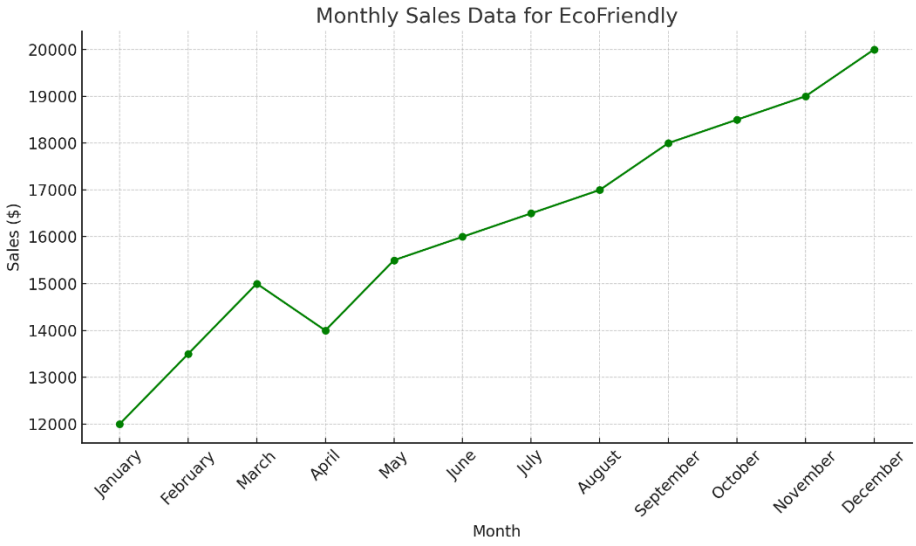
Detailed Implementation:

Introduction to Real-World Problem:

- **Example Script:** "Today, we'll explore how statistics help us understand the world around us. Specifically, we'll look at real-world data to understand the concepts of skewness, symmetry, and kurtosis."

Presenting the Problem:

- "Let's look at this graph showing the monthly sales data for EcoFriendly, a company that sells sustainable products. Notice the fluctuations over the year?"



- Encourage students to think critically by asking open-ended questions, "Looking at this sales data, what patterns do you notice? How might understanding the symmetry of this data help us predict future trends for EcoFriendly?"

- Sample Student Response: "From the sales data, I noticed a consistent upward trend in monthly sales, indicating that demand for EcoFriendly's sustainable products is increasing over time. The symmetry, or the steady growth pattern, suggests that we can expect sales to continue to rise if the current trend persists. This can help us predict future sales by projecting the growth rate forward."

Discussion and Engagement:

- After presenting the problem and encouraging initial observations, facilitate a group discussion. “How could statistical measures offer insights into data?”
    - Sample Student Response: "From the graph, I noticed that sales have been consistently increasing each month. This pattern suggests that there's a growing interest in sustainable products. By analyzing the trend and symmetry, we can predict that sales might continue to increase, especially during certain months. Understanding this could help EcoFriendly prepare better for high-demand periods."

Linking Concepts to Real-World Applications

- Conclude by explaining how skewness, symmetry, and kurtosis play crucial roles in data analysis and decision-making. "Understanding these concepts helps companies like EcoFriendly make informed decisions about product development, marketing strategies, and more."
- Understanding the shape of the data through skewness, symmetry, and kurtosis allows businesses to:
    - Make Informed Decisions: By analyzing the distribution of data, businesses can identify trends, risks, and opportunities that might not be apparent from mean or median values alone.
    - Manage Risks: Identifying skewness and kurtosis helps in understanding the likelihood and impact of extreme values, which is crucial for financial planning, insurance, and risk management.
    - Improve Operations: Symmetry in process-related metrics might indicate efficient operations, while deviations can highlight areas for improvement.
    - Customize Strategies: Different distributions may require different strategies. For instance, marketing strategies could be adjusted based on the skewness of customer spending patterns.

Explain (30 Minutes)

Begin the lesson with a bell ringer activity. Provide students with the QuantHub scavenger hunt for the resource What Does Variability, Skewness, and Kurtosis Describe About a Distribution?. Once they complete the scavenger hunt, begin the lesson.

Briefly explain the key concepts of skewness, symmetry, and kurtosis. Use slides or a whiteboard to present each concept clearly, employing visual aids such as diagrams or charts to illustrate.

Ensure to include examples that relate back to the real-world problem introduced during the Engage phase.

Detailed Implementation:

1. **Introduction to Key Concepts:**
    - Start with a brief overview: "Today, we're diving deeper into understanding data through the concepts of skewness, symmetry, and kurtosis."
    - Provide visuals of each concept by using the QuantHub Scavenger hunt Resource or by projecting or printing the images in the Appendix.
2. **Explaining Skewness:**
    - "Skewness measures the asymmetry of the data's distribution. Let's look at this chart showing a right-skewed distribution. Notice how most data points are gathered to the left and there is a long tail to the right?"
3. **Introducing Symmetry:**
    - "When data is symmetric, it means it's evenly spread around the center. Here's a symmetric distribution for comparison. It looks quite balanced, doesn't it?"
4. **Discussing Kurtosis:**
    - "Kurtosis tells us about the 'tailedness' of the distribution. A high kurtosis means more data points are in the tails. This graph illustrates a high kurtosis distribution."

Hands-On Activity

1. **Group Formation and Data Distribution:**
    - Divide students into small groups, providing each with the data set (See Appendix). "Let's split into groups and analyze different datasets to practice calculating our three key concepts."
2. **Guiding Through Statistical Software:**
    - Offer step-by-step instructions for using the statistical software. "First, let's load our data into the software. Now, find the function to calculate skewness..."
    - Be available to assist with questions and ensure every group is on track.

Group Discussion

1. **Sharing Findings**
    - Facilitate a discussion where each group shares their findings. "Group 1, what did you find about the skewness of your data? How does it reflect on the data's distribution?"
    - **Sample Student Response:** "Our data showed a right skew. This means there are a lot of lower values with some high values stretching the distribution to the right."

2. **Introducing Business Applications**

    - Risk Assessment: Skewness can indicate the presence of outliers or extreme values that could represent risks or opportunities. For example, a positively skewed distribution of product returns might highlight a few products with exceptionally high return rates, signaling quality or customer satisfaction issues.
    - Financial Analysis: In finance, the return distributions of assets are often skewed. Positive (right) skewness would imply a higher chance of unusually large gains, while negative (left) skewness indicates a higher risk of significant losses.
    - Fair and Balanced Processes: Symmetry can suggest that a process or system is functioning smoothly and predictably. For example, a symmetrical distribution of customer wait times might indicate efficient service with no unexpected delays.
    - Performance Analysis: Symmetrical distributions in performance metrics (e.g., sales across different regions) might suggest consistency and uniformity in operations, which is often a goal for businesses seeking to standardize quality and service.
    - Risk Management: High kurtosis in investment returns can signal a greater risk of extreme events, which is crucial for risk assessment and management strategies.
    - Quality Control: In manufacturing, kurtosis can help identify variability in product quality. High kurtosis might indicate the presence of defects or outliers in product dimensions or performance, necessitating adjustments in the production process.

Inspire (60 minutes)

Instruct students to create detailed diagrams of their analyzed data sets, including annotations for calculated measures of skewness, symmetry, and kurtosis.

Detailed Implementation:

Guidance for Selection

1. **Providing Resources:**
    - "Today, we'll embark on a real-world data analysis project. I've compiled a list of datasets from fields like finance, marketing, environmental science, and technology. Choose one that aligns with your career interests."
2. **Encouraging Informed Choices:**
    - Offer brief descriptions of how statistical analysis is used in each field, helping students make informed choices. (See Appendix for resource)
3. **Analysis and Interpretation (20 minutes):**
    - **Statistical Software Guidance:**

        - "Once you've selected your dataset, use the statistical software to calculate skewness, symmetry, and kurtosis. Create histograms or box plots to visualize the data's shape."
    - **Analytical Focus:**
        - Encourage critical thinking about the impact of these measures on data interpretation.

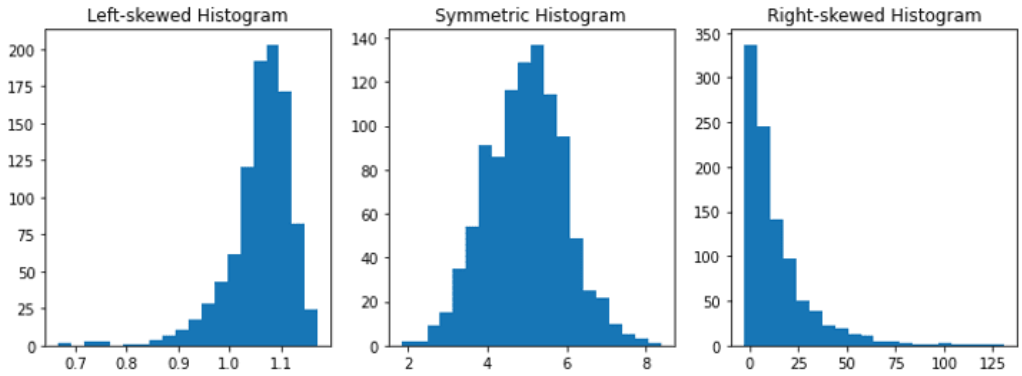
4. **Presentation and Reflection (15 minutes):**
    - **Preparing Presentations:**
        - Instruct students to prepare short presentations on their findings, emphasizing the implications of data shape in their selected field.
        - "In your presentation, explain how skewness, symmetry, and kurtosis provide insights into your dataset and how they might influence decision-making in your field."
    - **Peer Review Session:**
        - After presentations, conduct a peer review session. Encourage constructive feedback focusing on clarity, depth of analysis, and relevance (See Appendix for Learning Resource)

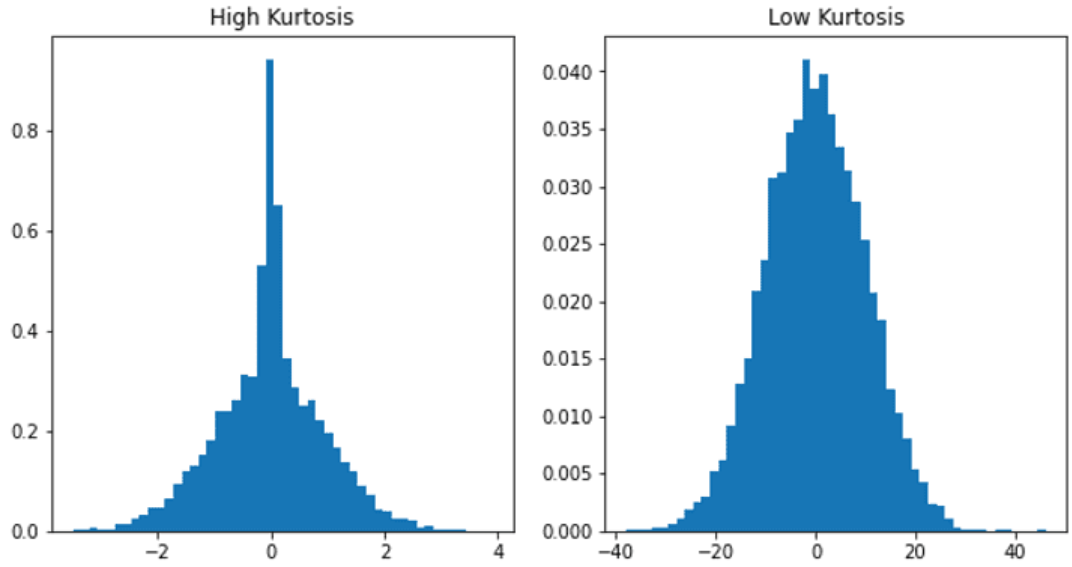
Assessment

- **Formative Assessment**: Histograms and Density Plots
- **Summative Assessment**: Collect students' diagrams and written summaries analyzing their data sets. Use the provided rubric to assess their ability to accurately interpret, summarize, and visualize the data's shape.

Appendix

Image Bank: Explain Phase





Case Study: Explain Phase

The Solar Panel Efficiency Project

Background

Mr. Thompson's high school physics class is conducting a month-long project to study the efficiency of solar panels under different conditions. They aim to understand how factors such as the time of day,

cloud cover, and the angle of the solar panels affect their power output. The class is divided into groups, and each group is responsible for recording the power output of their assigned solar panel at hourly intervals from 8 a.m. to 5 p.m. Additionally, they note the weather conditions and the angle of the panel to the sun. The project's goal is to gather data to analyze the efficiency of solar panels and understand how to optimize their use for renewable energy.

Data Collection
The students collected data over four weeks, resulting in a comprehensive dataset. For simplicity, the dataset below represents a summary of their findings, focusing on the total daily power output (measured in kilowatt-hours, kWh) of the solar panels under various conditions.

Data Set Summary
Total Days Recorded: 28 days
Measurement: Total daily power output in kWh

Day	Power Output (kWh)	Weather Condition	Panel Angle
1	4.5	Sunny	30°
2	4.2	Partly Cloudy	30°
3	3.8	Cloudy	30°
4	4.8	Sunny	45°
5	4.0	Sunny	15°
6	3.2	Cloudy	45°
7	4.7	Sunny	30°
...	...	...	...
28	3.5	Cloudy	15°

(Note: The table above is a simplified version of the dataset for the case study. Assume there are similar variations for the remaining days.)

Assignment
Analyze the dataset to identify patterns in solar panel efficiency based on the data in the chart above. Specifically, you will:

- Calculate the mean, median, and mode of the power output.
- Identify the skewness in the distribution of power outputs.
- Determine the symmetry or asymmetry in the data distribution.
- Analyze the kurtosis to understand the peakness of the data distribution.

Statistical Analysis and Careers Descriptions

1. **Finance:** In finance, statistical analysis is like using a magnifying glass to look closely at money matters. Imagine you have a piggy bank, and you want to know how much money you'll have in a year if you keep adding your allowance. By using past patterns, like how often you spend or

save, statistical analysis helps predict future money growth, decide where to invest your savings for more money, and even understand how the economy might affect your piggy bank.

2. **Marketing:** In marketing, it's all about understanding what people like and don't like. Think of it as being a detective at a school fair, trying to figure out which game booth is the most popular. By looking at data, like how many people visit each booth, what they buy, and what they say about it, marketers can figure out what games (or products) to make better, how to attract more people, and even predict what new game everyone will want to play next.
3. **Environmental Science:** Environmental scientists use statistical analysis like a tool kit to help the Earth. Imagine you're tracking the health of a forest near your school. By collecting data on things like the number of trees, types of animals living there, and the air quality, you can spot patterns and changes over time. This helps understand how human actions affect the environment, predict changes like global warming, and find ways to protect nature.
4. **Technology:** In the technology world, statistical analysis is used to make gadgets and apps smarter. Imagine your video game learns how you play and gets better at challenging you. By analyzing data from how users interact with software or gadgets, tech companies can improve products, make recommendations (like suggesting a new game you might like), and even develop new technologies that can predict what you'll want to do next.

Career Case Studies: Inspire Phase

Case Study 1: Finance- "Personal Savings Analysis"

Scenario:

In a study on personal savings, high school students were tasked with analyzing the savings behavior of individuals from various age groups. The data collected ranged from teenagers (starting at age 15) to elderly individuals (up to age 80), focusing on how much each person saved per month.

Data Set:

Age Range	Savings Range ($)	Most Common Savings ($)	Notes
15-20	$5-$200	$50	
21-30	$100-$500	$300	
31-40	$200-$700	$400	
41-50	$300-$800	$600	
51-60	$400-$900	$700	
61-70	$200-$700	$500	
71-80	$100-$600	$500	A few outliers saving above $1000

Concepts to Explore:

- **Skewness:** Identify which age groups have savings that are skewed left or right.
- **Symmetry:** Determine if any age group's savings distribution is symmetrical.
- **Kurtosis:** Analyze the peakedness or flatness of the savings distribution across different age groups.

Case Study 2: Marketing- "Social Media Engagement Rates"

Scenario:
A local business decided to analyze their social media engagement rates to improve their marketing strategy. The data includes likes, comments, and shares on posts across different platforms over a six-month period.

Platform	Likes Range	Comments Range	Shares/Retweets Range
Facebook	100-500	10-50	5-25
Instagram	200-1000	20-100	10-50
Twitter	50-250	5-25	2-15 (retweets)
LinkedIn	75-300	5-20	1-10

Concepts to Explore:

- **Skewness:** Evaluate the skewness in engagement rates across different platforms.
- **Symmetry:** Assess which platform has the most symmetrical engagement data.
- **Kurtosis:** Compare the kurtosis of engagement distributions to identify which is more peaked.

Case Study 3: Environmental Science- "Annual Rainfall Patterns"

Scenario:

Students were asked to analyze the annual rainfall patterns over the past decade in four different cities with diverse climates to understand trends and anomalies in environmental data.

Data Set:

City	Rainfall Range (inches)	Most Years Around (inches)	Notes
City A	20-40	30	
City B	10-70	-	A few years experiencing over 80 inches
City C	50-100	75	
City D	5-15	10	

Concepts to Explore:

- **Skewness:** Identify any cities with rainfall data that are significantly skewed.
- **Symmetry:** Analyze which city has the most symmetrical annual rainfall distribution.
- **Kurtosis:** Determine the kurtosis of rainfall data to find out which city shows extreme values or outliers.

Case Study 4: Technology- "Smartphone Battery Life"

Scenario:

A survey was conducted to find out the battery life of different smartphone brands used by high school students. The aim was to understand which brands offer longer battery life and how varied the battery life is among different models.

Data Set:

Brand	Battery Life Range (hours)	Most Models Around (hours)	Notes
Brand A	8-10	-	
Brand B	6-12	9	
Brand C	10-15	-	A few models up to 18 hours
Brand D	5-8	7	

Concepts to Explore:

- Skewness: Examine which brand has battery life data skewed to the left or right.
- Symmetry: Determine if any smartphone brand's battery life data is symmetrical.

- Kurtosis: Analyze which brand's battery life distribution is more peaked or has heavier tails.

How-To: Peer Review

Introduction: Peer reviewing is a way to help your classmates by giving them feedback on their work. It's like being a helpful friend who points out both the good things and the areas where they can improve. Here's a step-by-step guide on how to do it:

Step 1: Read Carefully

- Start by reading your peer's work carefully. Take your time to understand what they're trying to say.

Step 2: Note the Good Stuff

- Begin your review by mentioning what you liked about their work. Did they have a strong introduction? Were their ideas clear and easy to follow? Be specific and positive.

Step 3: Point Out Areas for Improvement

- After the good stuff, it's time to help them get even better!
- Be kind and constructive. Instead of saying, "This part is bad," you can say, "This part could be clearer if..."
- Look for things like grammar mistakes, confusing sentences, or places where they could add more details.

Step 4: Ask Questions

- If you didn't understand something or if they left you with questions, don't hesitate to ask them. For example, "I'm not sure what you mean here. Can you explain it differently?"

Step 5: Make Suggestions

- Offer suggestions for improvement. If you have ideas for how they can make their work stronger, share them. For instance, "You could add more examples to support your point."

Step 6: Be Respectful

- Remember to be respectful and kind in your comments. Your goal is to help, not hurt.

Step 7: Proofread

- Before you finish, read through your own comments to make sure they're clear and make sense. You want your feedback to be helpful.

Step 8: Summarize Your Review

- In the end, sum up your feedback with a positive note. For example, "Overall, your work is good, and with a few changes, it can be even better!"

Conclusion: Peer reviewing is a valuable skill that helps both you and your classmates grow as writers and learners. By giving and receiving feedback, you all become better at communicating your ideas and improving your work. So, don't be shy—give it a try and help your friends succeed!

Teacher Rubric: Assessment

Criteria	Excellent (5)	Good (4)	Satisfactory (3)	Needs Improvement (2)	Unsatisfactory (1)
Dataset Selection	Student chose a dataset aligned with their career interests, demonstrating a clear understanding of how it relates to the field.	Student chose a dataset related to their career interests but with some lack of clarity on its relevance.	Student selected a dataset, but its alignment with their career interests is not evident.	Student selected a dataset that is not related to their career interests.	Student did not select a dataset.
Statistical Software Usage	Student effectively used statistical software to calculate skewness, symmetry, and kurtosis, accurately representing the data's shape.	Student used statistical software to calculate measures, but with minor errors or inaccuracies in data representation.	Student attempted to use statistical software but made significant errors in calculating measures or data representation.	Student used statistical software with major errors in calculation and data representation.	Student did not use statistical software.
Analytical Focus	Student demonstrated a deep understanding of how skewness, symmetry, and kurtosis impact data interpretation in their chosen field, providing	Student showed a good understanding of the impact of these measures on data interpretation, providing adequate analysis.	Student discussed the impact of measures but with limited depth or insight into their field's context.	Student mentioned the measures' impact but lacked clear analysis or connections to their field.	Student did not discuss the impact of measures on data interpretation.
Presentation Clarity	The presentation was well-structured, clear, and concise, effectively communicating findings and insights.	The presentation was mostly clear and well-organized, with minor issues in communication.	The presentation had some clarity and organization issues, making it challenging to follow in parts.	The presentation lacked clear structure and organization, leading to confusion.	The presentation was disorganized and incoherent.
Relevance to Chosen Field	Student effectively explained how skewness, symmetry, and kurtosis provide insights into their dataset and influence decision-making in their field.	Student provided a reasonable explanation of the measures' relevance to their dataset and field.	Student mentioned the measures' relevance but with limited detail or explanation.	Student briefly mentioned relevance without clear explanations.	Student did not address the relevance of measures to their field.
Visual Representation Quality	Student created detailed diagrams with accurate annotations for skewness, symmetry, and kurtosis, enhancing understanding.	Student created diagrams with annotations but with minor inaccuracies or omissions.	Student attempted to create diagrams and annotations but with significant errors or lack of detail.	Student's diagrams and annotations were rudimentary and contained major inaccuracies.	Student did not create any diagrams or annotations.
Overall Presentation Skills	Student presented confidently, engaging the audience, and effectively answering questions.	Student presented with confidence, responding to questions but with minor hesitations.	Student presented adequately but lacked some confidence and had difficulty answering questions.	Student presented with noticeable hesitation and difficulty in responding to questions.	Student's presentation was unconfident, with frequent hesitations and inability to answer questions.
Overall Assessment	The student excelled in all aspects, demonstrating a deep understanding of the activity's goals.	The student performed well in most aspects, with minor room for improvement.	The student met the basic requirements of the activity but had several areas that could be improved.	The student had significant shortcomings in multiple areas of the activity.	